LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

UMT 3501 – ABSTRACT ALGEBRA

Date: 08-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**MCQ**
a)	The union of the two sets A and B is a) $\{x \vee x \in A \wedge x \in B\}$ b) $\{x \vee x \in A \vee x \in B\}$ c) $\{x \vee x \in A \wedge x \notin B\}$ d) $\{x \vee x \notin A \wedge x \notin B\}$
b)	If G is a finite group and $a \in G$, then $a^{o(G)} = ¿$ a) 0 b) $-e$ c) e d) ∞
c)	The product of even and odd permutations is a) an even permutation b) an odd permutation c) neither even nor odd permutation d) an identity permutation
d)	The set of all rational numbers under the usual addition and multiplication of rational numbers is a) a ring alone b) a commutative ring with unit element c) not a field d) not a ring
e)	Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then a) $d(a) = d(b)$ b) $d(a) \neq d(b)$ c) $d(a) > d(b)$ d)$d(a) < d(b)$
2.	**Fill in the blanks**
a)	If for every $a, b \in G$ such that $a.b = b.a$, then a group G is said to be ....
b)	HK is a subgroup of G if and only if $HK = ¿$.................
c)	If ϕ is $1-1$, then a homomorphism ϕ from G into $\overline{G}$ is known as ....
d)	If ϕ is a homomorphism of R into R', then $\phi(0) = ¿$................
e)	The process of transforming a plain text message to a cipher text message is ....
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	The set $G = \{1, -1\}$ is a group under the multiplication of real numbers.
b)	A subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is a left cost of N in G.
c)	Let G be the group of integers under addition and let $\overline{G} = G$. Then ϕ is a homomorphism which is defined by $\phi(x) = x^2$ for the integer $x \in G$.
d)	A field is not a commutative division ring.
e)	If R be a commutative ring with unit element whose only ideals are $\{0\}$ and R itself, then R is a field.
4.	**Answer the following**
a)	When do we say that the two integers a and b are relatively prime?
b)	Define a quotient group of G /N.
c)	What do we mean by an automorphism of a group G?

d)	Define an ideal U of a ring R.
e)	When do we say an ideal $M \neq R$ in a ring R to be an maximal ideal of R?
SECTION B - K3 (CO2)	
Answer any TWO of the following in 100 words each. (2 x 10 = 20)	
5.	Using the axioms of group, check that $G=S_3$, the set of all $1-1$ mappings from the set $\{1,2,3\}$ onto itself, is a group under the composition of mappings.
6.	State and prove Lagrange's theorem.
7.	If U is an ideal of the ring R, prove that R/U is a ring and is a homeomorphic image of R.
8.	Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d. Also prove that $d=\lambda a+\mu b$ for some $\lambda, \mu \in R$.
SECTION C – K4 (CO3)	
Answer any TWO of the following in 100 words each. (2 x 10 = 20)	
9.	Verify that the relation $a \equiv b \bmod H$ is an equivalence relation for a subgroup H of a group G.
10.	Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K. Establish that $G/K \approx \overline{G}$.
11.	Prove that if R is a ring then for all $a, b \in R$, i. $a0=0a=0$, ii.$a(-b)=(-a)b=-(ab)$, and iii. $(-1)a=-a$, if in addition R has a unit element 1.
12.	Explain Private key cryptography in detail.
SECTION D – K5 (CO4)	
Answer any ONE of the following in 250 words (1 x 20 = 20)	
13.	Prove that the following statements for a group G: a. The identity element of G is unique, b. Every $a \in G$ has a unique inverse in G, c. For every $a \in G$, $(a^{-1})^{-1}=a$, and d. For all $a, b \in G$, $(a.b)^{-1}=b^{-1}.a^{-1}$.
14.	i. State and prove Cayley's theorem. (12 marks) ii. Prove that a finite integral domain is a field. (8 marks)
SECTION E – K6 (CO5)	
Answer any ONE of the following in 250 words (1 x 20 = 20)	
15.	i. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively, prove that $o(HK)=\frac{o(H)o(K)}{o(H\cap K)}$. (12 marks) ii. Prove that a subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G. (8 marks)
16.	Prove the Statement: "If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is a field".

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